Section 4.1

Section 4.1

(46) Classify the polynomial $f(x) = 1 - 3x^2 - 5x^4$ as constant, linear, quadratic, cubic or quartic and determine the leading term, leading coefficient and degree.

46

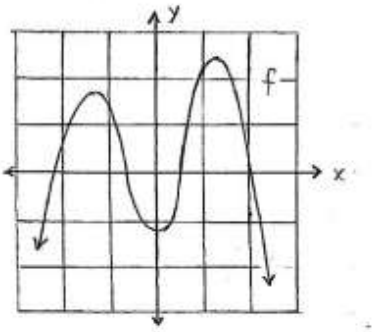
The polynomial $f(x) = 1 - 3x^2 - 5x^4$ is **quartic**.

Its leading term is $-5x^4$ and the leading coefficient is -5

The degree is **4**

47

(47) Use the graph of f to identify the type and estimate the values of the extrema.



The following values are approximations only.

The function has a local maxima at the points (-1.3, 1.75) and (1.3, 2.5) and a local minimum at the point (0, -1.25).

Therefore,

local maximum value: 1.75 and 2.5 (maximum value)

local minimum value: -1.25

48

(48) Determine if $f(x) = x^5 - 13x^3 + 36x$ is even, odd or neither. Justify your answer.

We evaluate $f(-x)$ and compare it with $f(x)$ or $-f(x)$

We have

$$f(-x) = (-x)^5 - 13(-x)^3 + 36(-x)$$

$$= -x^5 + 13x^3 - 36x$$

$$= -(x^5 - 13x^3 + 36x)$$ Note: The expression inside the parenthesis is equal to f(x)

$$= -f(x)$$

Since $f(-x) = -f(x)$, then it is an **odd function**

49

(49) Prove symbolically that $f(x) = \sqrt{1-x^2}$ is even.

We evaluate $f(-x)$ and compare it with $f(x)$ or $-f(x)$

We have

$$f(-x) = \sqrt{1-(-x)^2}$$

$$= \sqrt{1-x^2}$$

$$= f(x)$$

Since $f(-x) = f(x)$, then it is an **even function**

50

(50) Complete the table assuming f is an odd function.

x	-6	-4	-2	0	2	4	6
f(x)	0	10			-8		

Since f is an odd function, then $f(-x) = -f(x)$.

It is also true that $f(x) = -f(-x)$ for odd functions.

Therefore,

$f(-2) = -f(-(-2)) = -f(2) = -(-8) = 8$ Note: From the given table $f(2) = -8$

$f(4) = -f(-4) = -(10) = -10$ Note: From the given table $f(-4) = 10$

$f(6) = -f(-6) = -(0) = 0$ Note: From the given table $f(-6) = 0$

Using $f(-x) = -f(x)$, we get $f(-0) = -f(0)$ which is also the same as $f(0) = -f(0)$

To make the equation true, $f(0) = 0$

Below is the complete table

x	-6	-4	-2	0	2	4	6
f(x)	0	10	**8**	**0**	-8	**-10**	**0**

Section 4.2

Section 4.2

(51) Describe the end behavior (graphically and symbolically) of Exercise 46. Justify your answer.

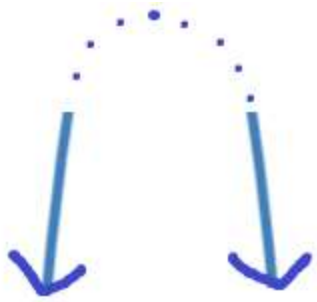
51

The function is $f(x) = 1 - 3x^2 - 5x^4$

Since the leading coefficient -5 which is a negative number and the degree is 4 which is even, then the end behavior symbolically is

As $x \to -\infty$, $f(x) \to -\infty$ (left side will fall to $-\infty$) and as $x \to \infty$, $f(x) \to -\infty$ (right side will fall to $-\infty$)

Graphically,



52

(52) In Exercise 47, (a) approximate the coordinates of the turning points, (b) estimate the x-intercepts and (c) state the minimum degree of f.

Part-a

The following values are approximations only.

The function has a local maxima at the points (-1.3, 1.75) and (1.3, 2.5) and a local minimum at the point (0, -1.25). Therefore, the points (-1.3, 1.75), (0, -1.25) and (1.3, 2.5) are the turning points

Part-b

The x-intercepts are approximately (-2.25, 0), (-0.75, 0), (0.5, 0), and (2, 0)

Part-c

(c) state the minimum degree of f.

Since there are 4 x-intercepts, then the minimum possible degree is **4**

53

(53) Assuming the table in Exercise 50 reflects a data sampling, (a) make a line graph of the data, (b) state the minimum degree of f, (c) decide if $a > 0$ or $a < 0$ and (d) find a formula for f.

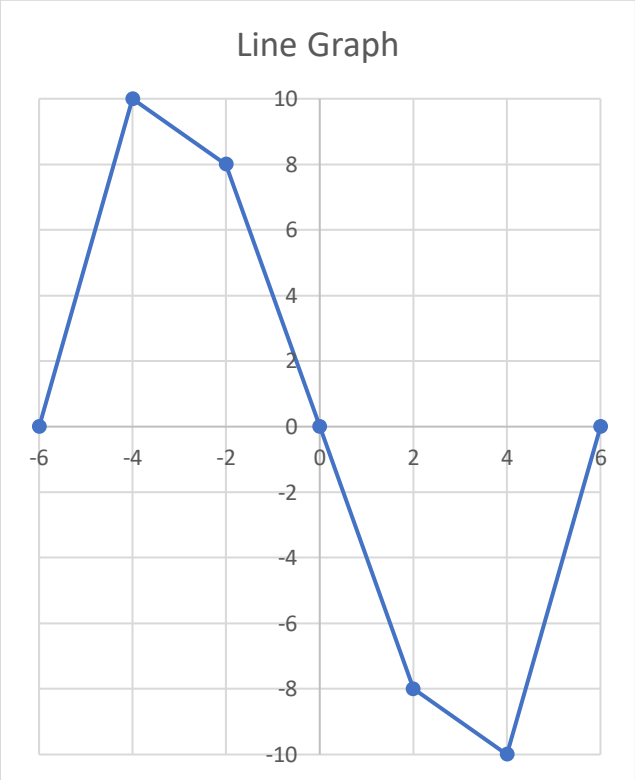
Part-a

From Exercise 50, the complete table is

x	-6	-4	-2	0	2	4	6
f(x)	0	10	8	0	-8	-10	0

Part-a

We plot the points (-6, 0), (-4, 10), (-2, 8), (0,0), (2, -8), (4, -10), and (6, 0) and connect them by line segments.



Part-b

Since there are 3 x-intercepts, the minimum possible degree of f is **3**

Part-c

Since the right side increases, then a > 0

Part-d

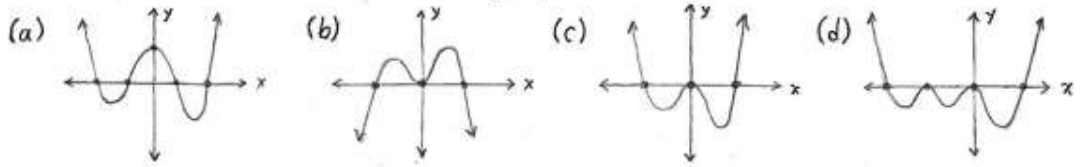
According to the calculator, the cubic function that best models the data is

$$f(x) = 0.125x^3 - 4.5x$$

and it exactly fits the data

54

(54) Use turning points, the leading-term test and/or x-intercepts to match $f(x) = 2x^4 - 6x^2 - x$ with its graph (a-d). Justify your answer.



The leading term is $2x^4$ and the leading coefficient is 2 which is positive.

The degree is 4 which is an even number.

Therefore, the end behavior is that the left side and right side of the graph both rises.

We must eliminate option “b” because its end behavior is that the left side and right side of the graph both falls.

The degree is 4 so the maximum number of turning points must be n-1 = 4-1 = 3 o we eliminate option “d” which has 5 turning points.

$f(0) = 2(0)^4 - 6(0)^2 - 0 = 0$ so the graph must contain the point (0, 0) and we eliminate option “a” because it does not contain the point (0, 0)

The option left is “c” and this is our final answer.

Answer: **“C”**

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Section 4.2 (cont'd)

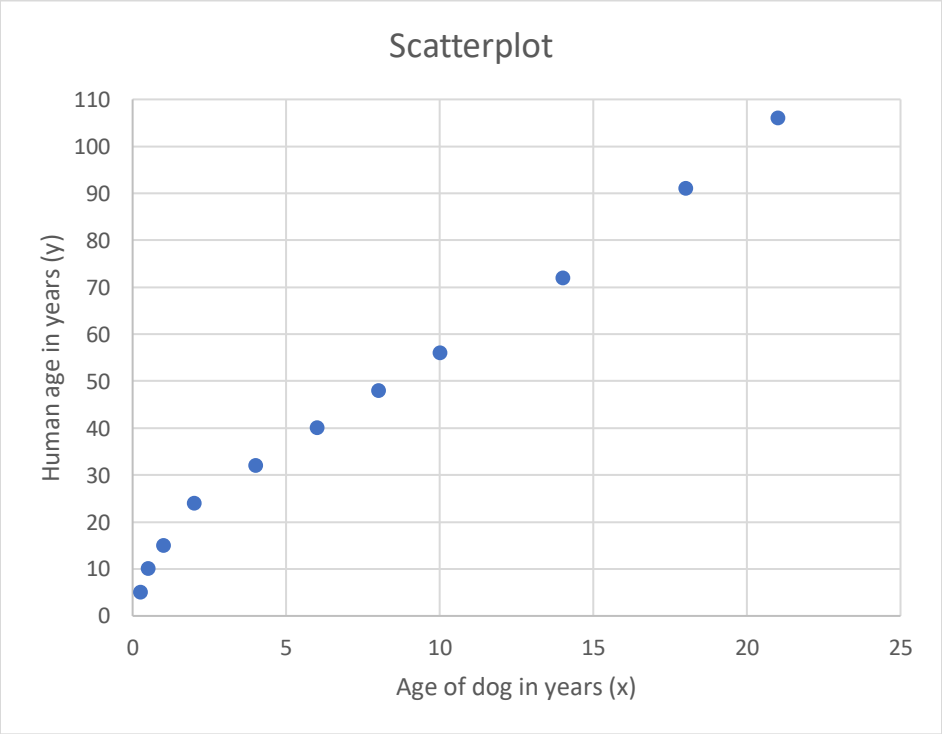
(55) The following table lists the age equivalents for dogs and humans.

Age of Dog (in years)	x	0.25	0.5	1	2	4	6	8	10	14	18	21
Human Age (in years)	h(x)	5	10	15	24	32	40	48	56	72	91	106

(a) Use your calculator to find a linear, quadratic or cubic function that best models the data. Write down and justify your answer.

Part-a

We choose a linear function because the points approximately follow a straight line. See the scatterplot below where we plot the points.



According to the calculator, the best linear function is

$h(x) = 4.55x + 10.31$

Part-b

(b) Use your model to estimate the equivalent human age for a dog that is 2 years old.

Using the linear model $h(x) = 4.55x + 10.31$, we have

$h(2) = 4.55(2) + 10.31$

= 19.41 years